Time Series Analysis

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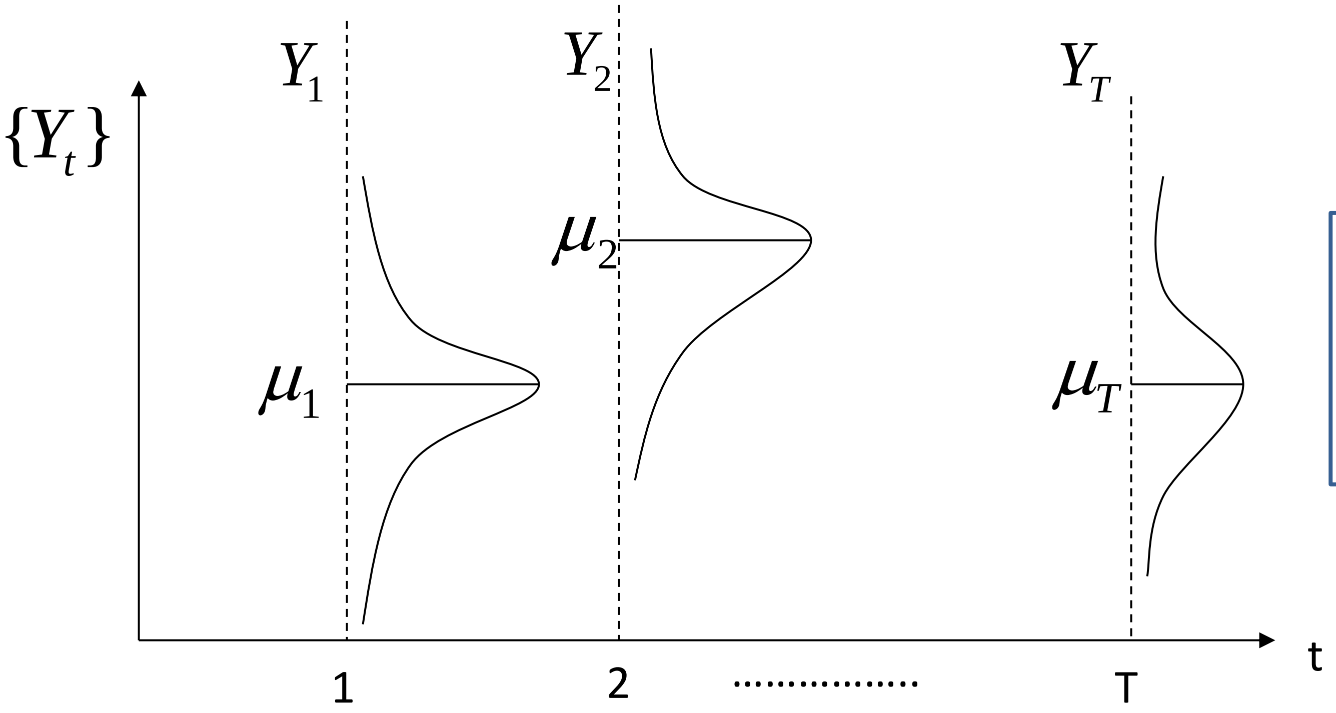
What is a Time Series?

- Def: Stochastic Process

$\{Y_t\} = \{Y_1, Y_2, \ldots Y_T\}$ = Collection of random variables.

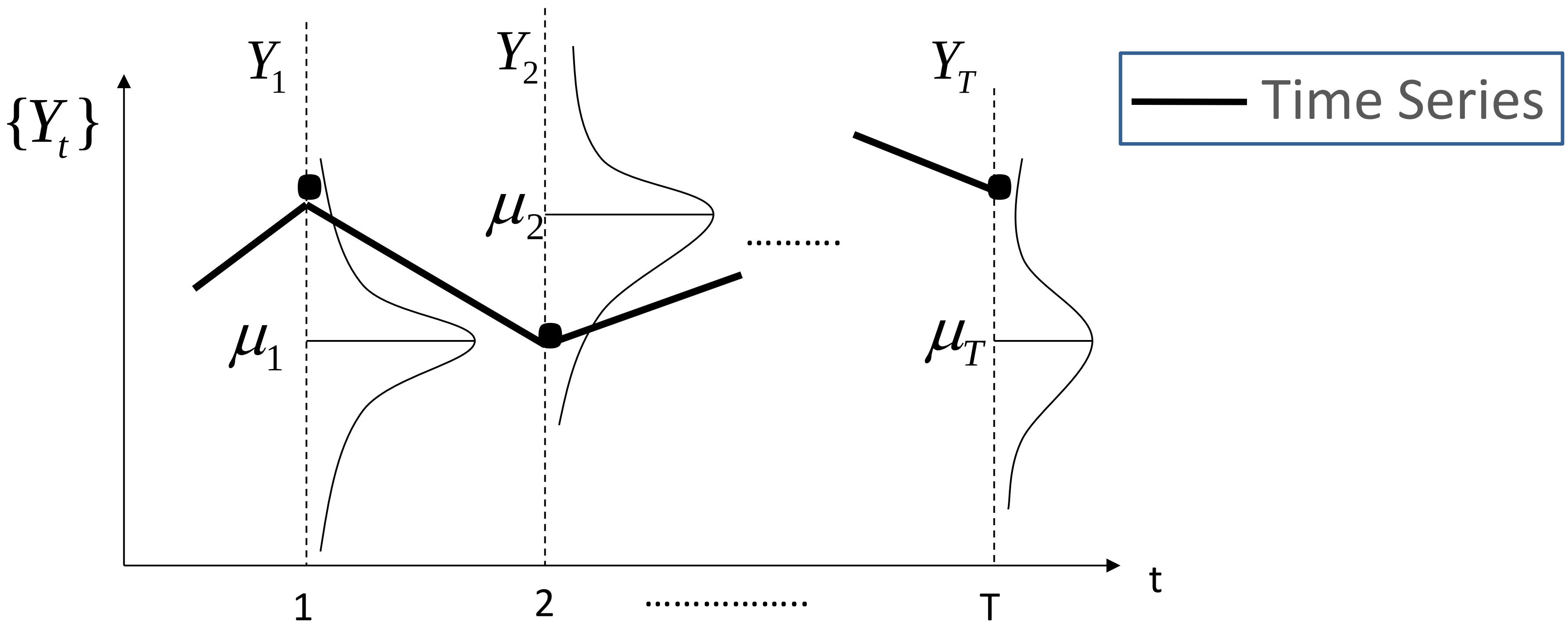


Each R.V can have a different μ, σ, and pdf.

What is a Time Series?

- **Def**: Time Series

  $\{y_t;\ t=1,2,\ldots\} = \{y_1, y_2, \ldots y_T\}$ = Sample realization of a stochastic process.

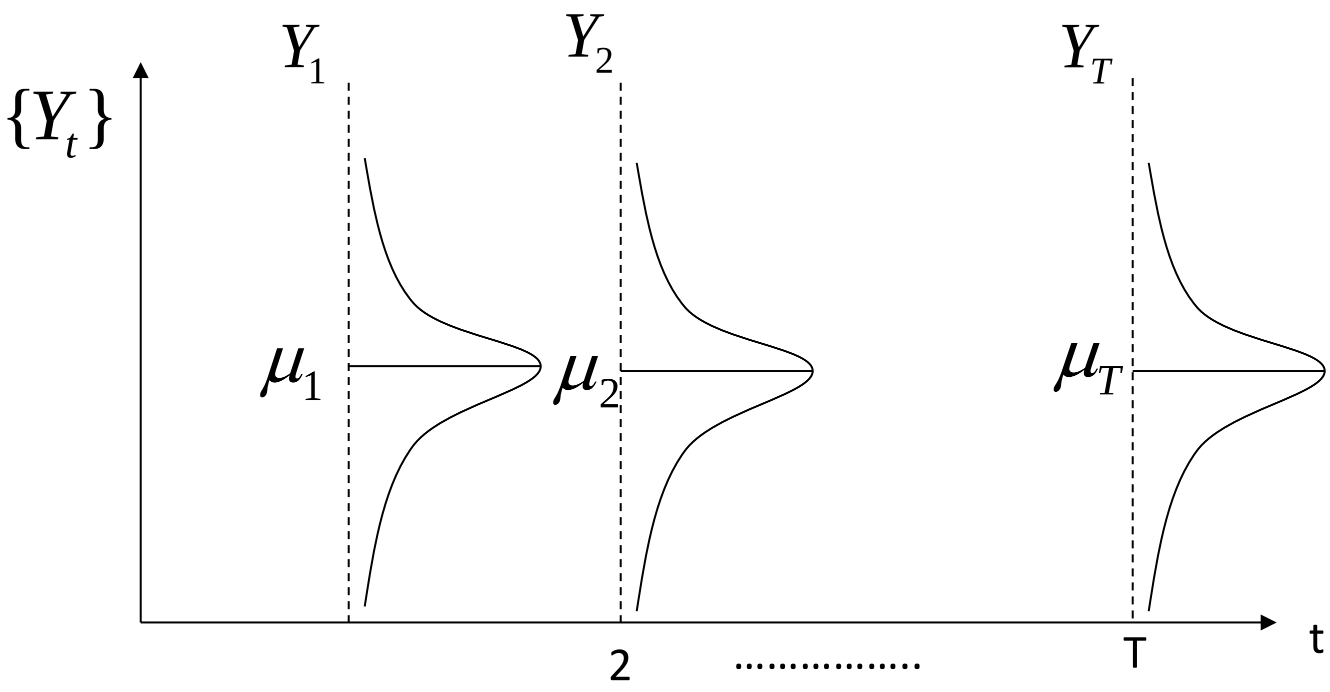


Stationarity

- Def: First Order Strongly Stationary

= All R.Vs have the same pdf's (all moments are the same). $f_{Y_1}(y) = f_{Y_2}(y) = \cdots = f_{Y_T}(y)$

→

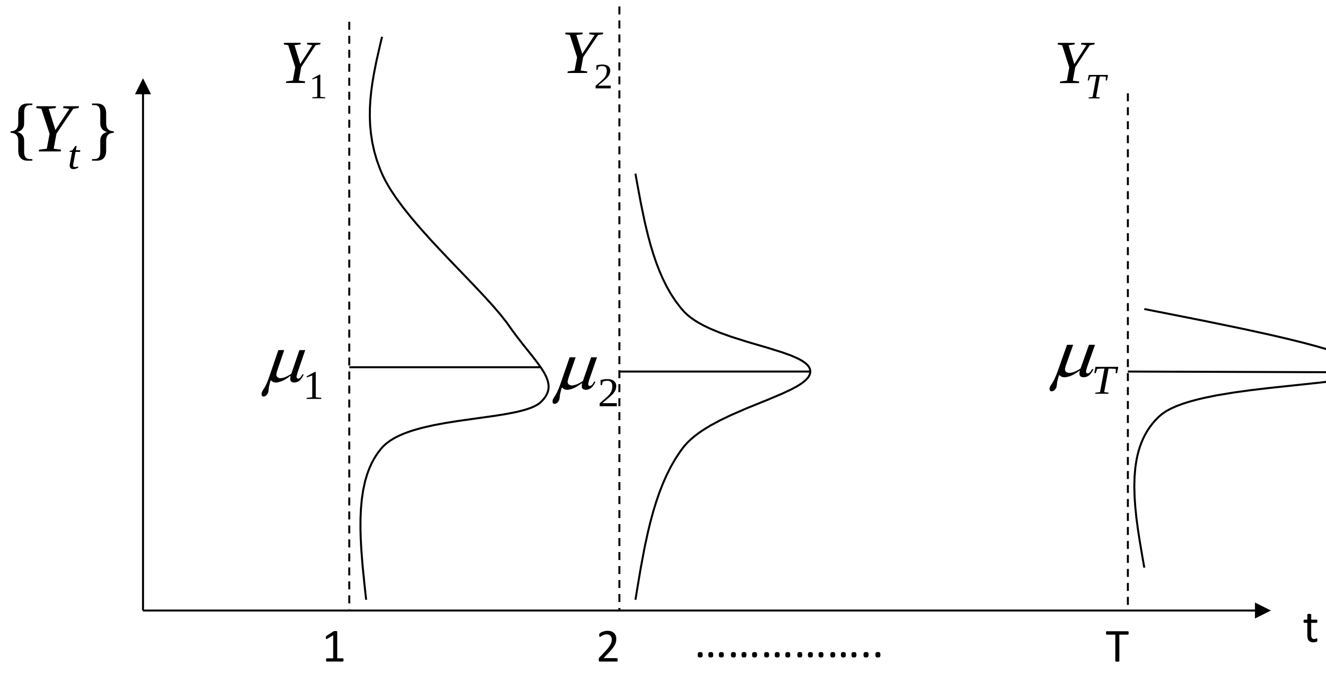


Stationarity

- Def: First Order Weakly Stationary

  = All R.Vs have the same means.

  $\rightarrow \mu_{Y_1} = \mu_{Y_2} = \cdots = \mu_{Y_T}$



Stationarity

- Def: Second Order Weakly Stationary
  = *Covariance Stationary*

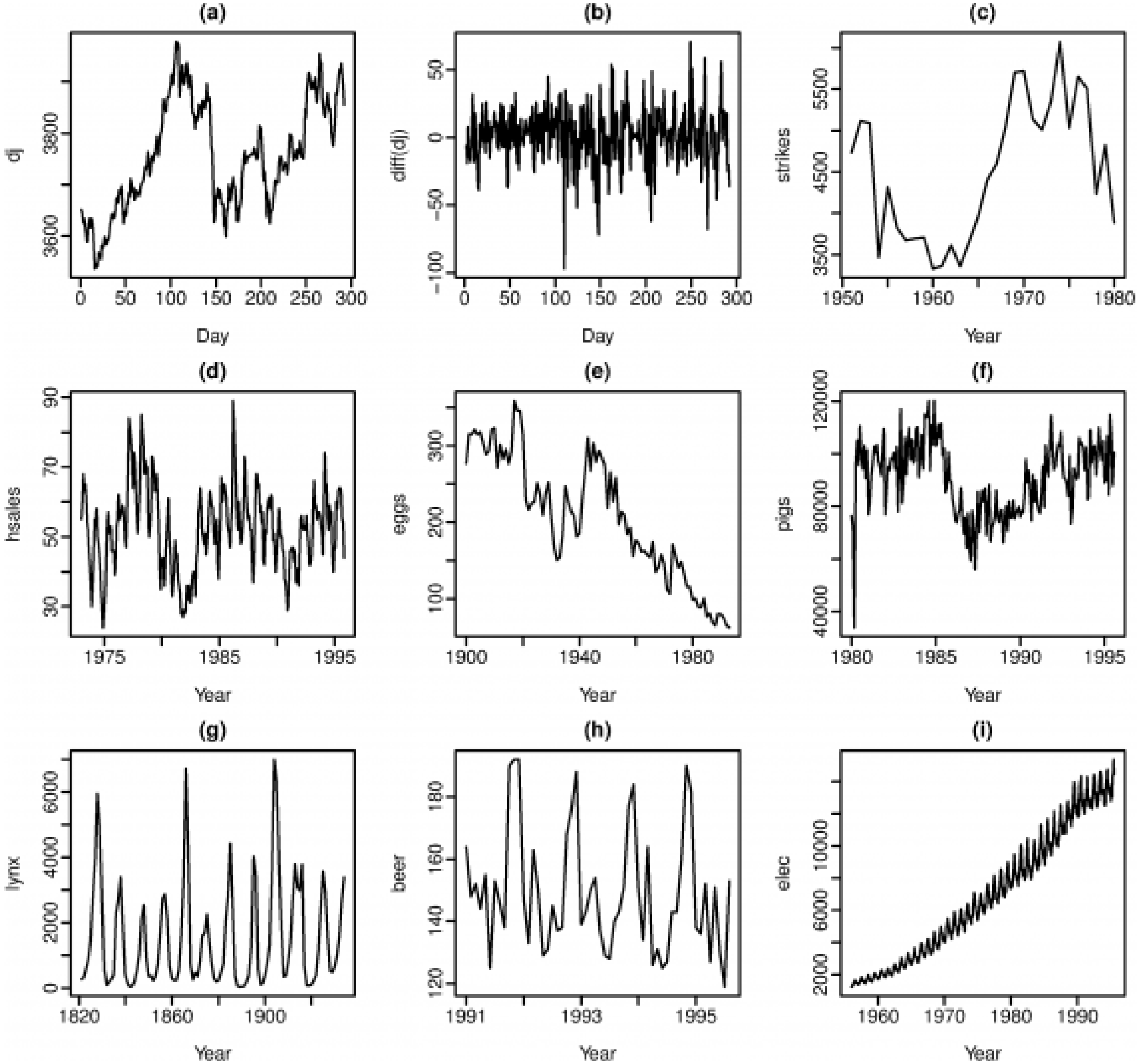
i. Means: $\mu_{Y_1} = \mu_{Y_2} = \cdots = \mu_{Y_T}$

ii. Variances: $\sigma^2_{Y_1} = \sigma^2_{Y_2} = \cdots = \sigma^2_{Y_T} = \sigma^2$

iii. Time Independent Covariances: $\rho_{Y_t, Y_{t-k}} = \rho_{|k|}$

The strength of the linear association between the two R.Vs only depends on how many periods (k) apart they are.

Stationarity

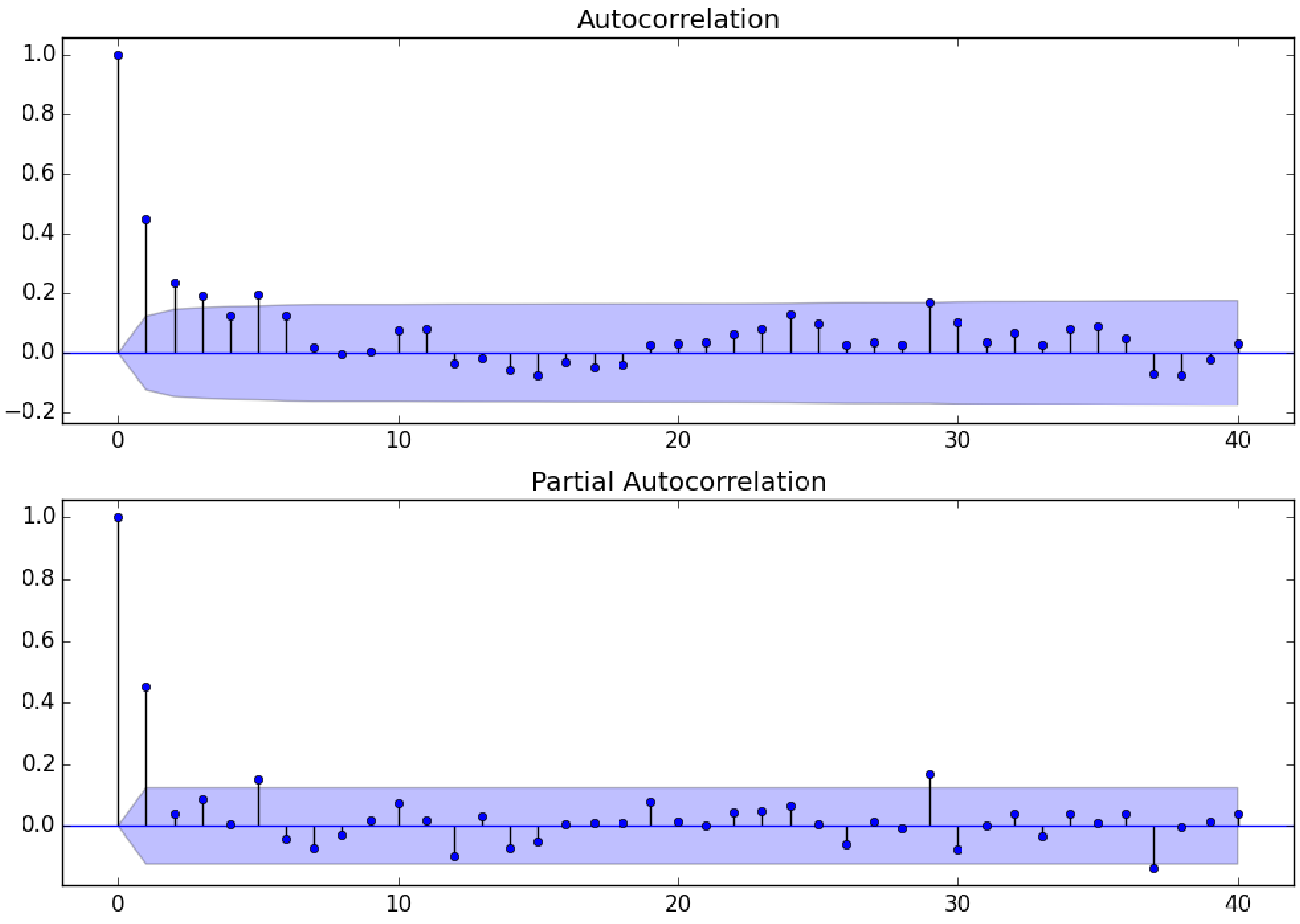


The Autocorrelation Functions

- Autocovariance Function (ACF): Measures correlation between time series and lagged versions of itself

- Partial Autocorrleation Function (PACF): Measures correlation between time series and lagged versions of itself accounting for all lags in between

The Autocorrelation Functions



Moving Average Models

- Def: MA(q) = Moving Average process of order q≥0: $Y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q}$,
- Examples:
  - MA(1): $Y_t = \mu + \theta_1 \varepsilon_{t-1} + \varepsilon_t$
  - MA(5): $Y_t = \mu + \theta_1 \varepsilon_{t-1} + \theta_5 \varepsilon_{t-5} + \varepsilon_t$
  - MA(10): $Y_t = \mu + \theta_{10} \varepsilon_{t-10} + \varepsilon_t$

Wold's Theorem (Part I)

- Wold's Representation Theorem: Let $\{y_t\}$ be any zero-mean covariance-stationary process. Then

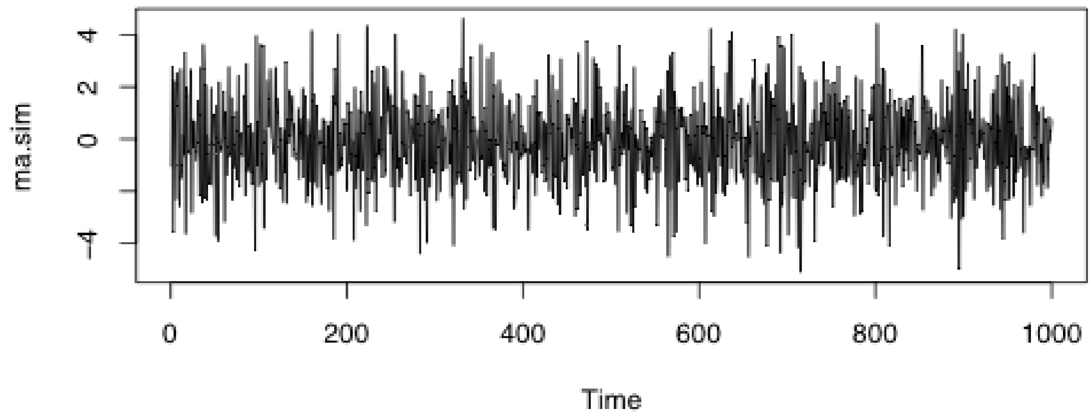
$$y_t = B(L)\varepsilon_t = \sum_{i=0}^{\infty} b_i \varepsilon_{t-i} \qquad \varepsilon_t \sim WN(0, \sigma^2)$$

(Innovations)

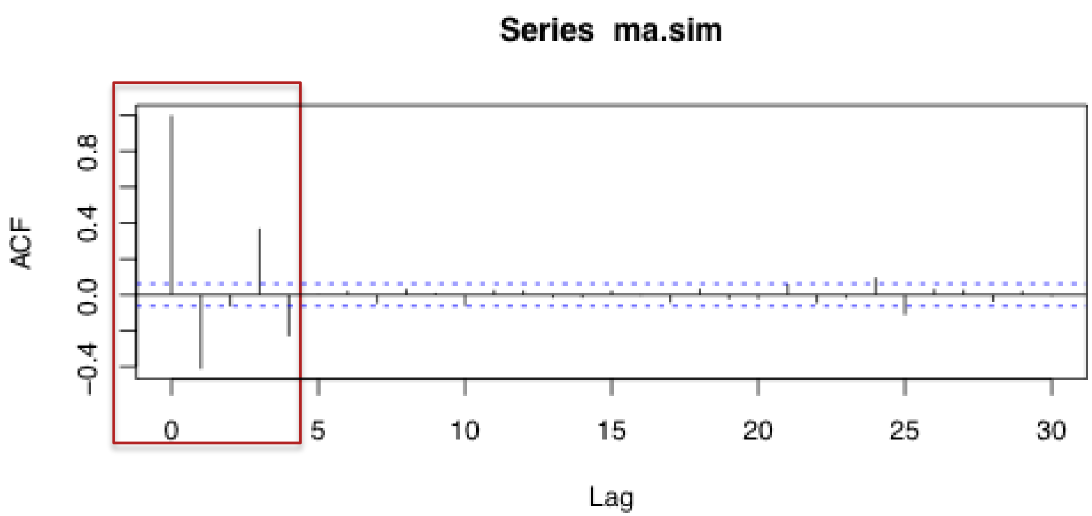
Moving Average Models

Example: MA(q) Process

What type of a process is this?



MA(4) Process



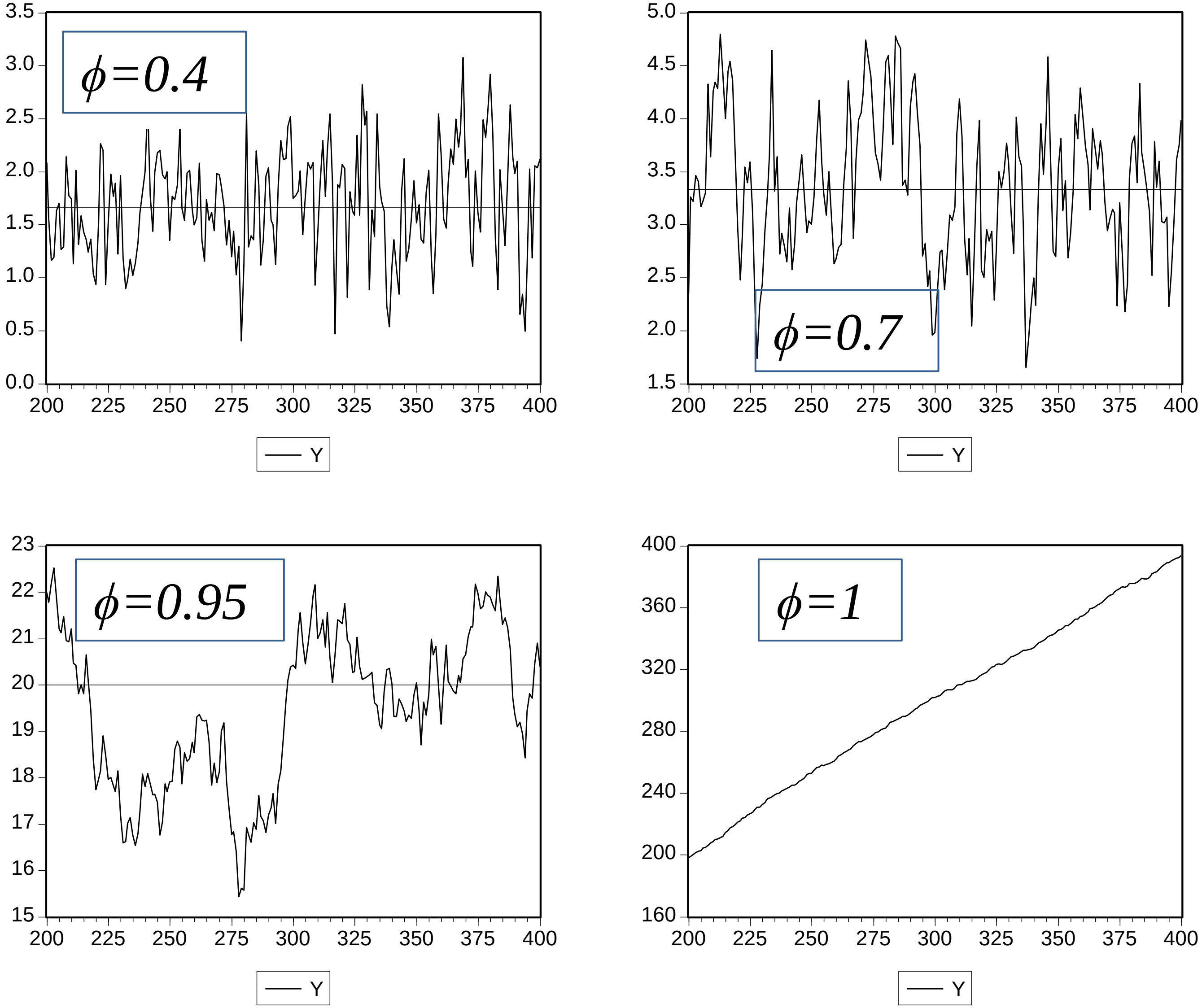
In general, for any MA(q) process, $\rho_k = 0$ for any $k>q$.

Autoregressive Models

- Def: AR(p) = Autoregressive process of order p≥0: $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_q Y_{t-p} + \varepsilon_t$, where $\varepsilon_t \sim (0, \sigma^2)$, and φ = persistence parameter.

- Examples:
  - AR(1): $Y_t = c + \phi_1 Y_{t-1} + \varepsilon_t$
  - AR(2): $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t$
  - AR(6): $Y_t = c + \phi_6 Y_{t-6} + \varepsilon_t$

AR(1): $Y_t = c + \phi Y_{t-1} + \varepsilon_t$



Unit Roots: Estimation and Testing
Unit Root Tests

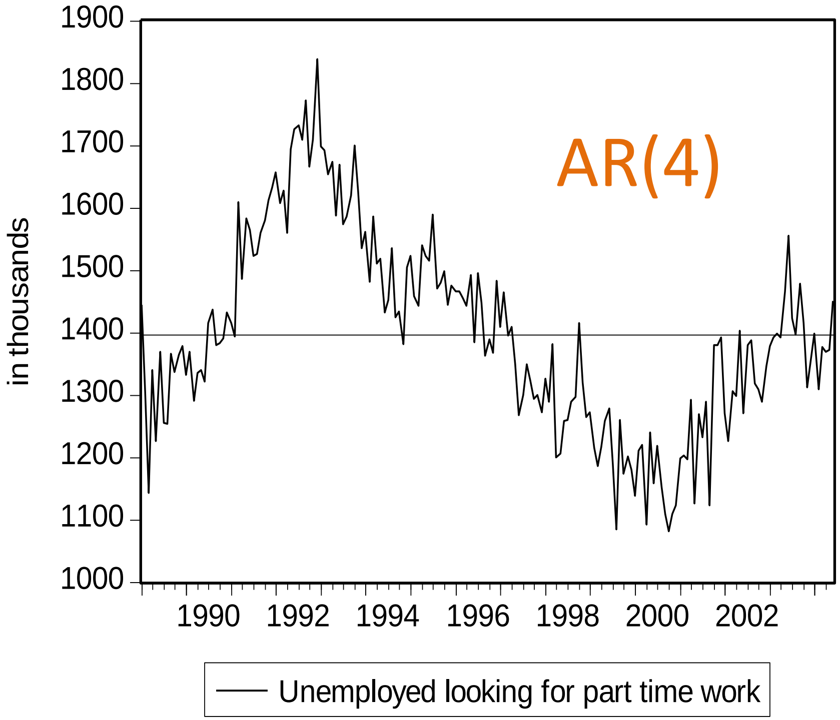
- Series with unit roots, should be checked for their presence via e.g., a t-statistic

$$\hat{\tau} = \frac{\hat{\varphi} - 1}{s\sqrt{\frac{1}{\sum_{t=2}^{T} y_{t-1}^2}}}$$ (Dickey-Fuller Statistic)

- For the unit root case, $\hat{\tau}$ follows a Dickey-Fuller Distribution.
- For the general, nonzero mean case (under the alternative hypothesis), the process is a covariance stationary AR(1) process in deviations from the mean. → $y_t = \alpha + \varphi\, y_{t-1} + \varepsilon_t$, where $\alpha = \mu(1-\varphi)$.
- Note: The Dickey-Fuller statistic table is for (α, φ)=(0,1).

Autoregressive Models
Example: AR(p) Process

Q: What type of a process is this?



Sample: 1989:01 2004:06
Included observations: 186

Lag	AC	PAC
1	0.882	0.882
2	0.874	0.434
3	0.851	0.176
4	0.841	0.136
5	0.829	0.094
6	0.803	-0.030
7	0.796	0.040
8	0.775	-0.009
9	0.766	0.024
10	0.737	-0.072
11	0.712	-0.084
12	0.672	-0.158
13	0.676	0.098
14	0.638	-0.076
15	0.623	0.002
16	0.586	-0.086
17	0.553	-0.096
18	0.539	0.009
19	0.508	0.002
20	0.482	-0.041

In general, for any AR(p) process, $p_k = 0$ for any $k>p$.

Autoregressive Moving Average (ARMA) Models→ARMA(p,q)

- The ARMA(1,1) Process: $Y_t = \phi Y_{t\text{-}1} + \theta \varepsilon_{t\text{-}1} + \varepsilon_t$
- The ARMA(p,q) Process:

$$Y_t = \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q} + \varepsilon_t$$

Mystery Process